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RESEARCH MEMORANDUM

SIMPLIFIED THEORY FOR DYNAMIC RELATION OF RAMJET PRESSURES AND FUEL FLOW

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

RESEARCH MEMORANDUM

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SUMMARY

An analysis is made to determine, for control design purposes, the approximate response of pressures in the ramjet engine to changes in fuel flow. The problem is simplified by separating it into two parts. In the first part, the response after dead time is treated by the linearized lumped-parameter method. Consideration is limited to ramjet operation that is critical or supercritical. In the second part, the dead time occurring between the change in fuel flow and the beginning of the pressure response is discussed.

Comparisons of the theory with experimental data indicate that the response after dead time can be approximated by a second-order lead-lag and that the dead time can be calculated on the basis of a sonic disturbance propagating through steady flow. The time constants of the theoretical lead-lag depend on the dynamic effects of the diffuser terminal shock and on the ratios of the significant volumes to the steady-state volume flow rate.

INTRODUCTION

Recent investigations have indicated that the most suitable controls for the supersonic ramjet engine are those that use a pressure, or group of pressures, within the engine as the controlled variable. In most ramjets the pressure will be controlled by means of fuel-flow manipulation. A method to predict the dynamic relation between these two variables is of great interest, therefore, in the synthesis of ramjet control systems.

When considered in rigorous form, the problem posed by this interest is a formidable one. It is concerned with the propagation, reflection, and interaction of pressure waves and with the complex flow patterns built up by these waves. The variables of such flows are, of course, functions of both time and position, and the partial differential equations relating them are highly nonlinear. Even when a flow can be treated as one dimensional, the nonlinearity of the governing equations confronts any attempt to obtain a general solution. In fact, there seems to be no convenient way to work with the rigorous problem apart from stepwise numerical calculation of each transient. The most convenient approach to these numerical calculations is, perhaps, the method of characteristics, which is well discussed in the literature (e.g., refs. 1 and 2).

A characteristics solution may not be warranted, however, for the purpose of synthesizing ramjet controls. Such a solution is very tedious and time consuming, and its potential accuracy will not be realized unless the initial conditions of the gas flow are well defined throughout the ramjet. Usually such detailed information is not available. Furthermore, numerical calculations must be interpreted if an understanding of the general problem is desired, and unfortunately they tend to hide any properties common to all transients.

With certain stipulations to ensure linearity, of course, the fundamental equations of transient gas flow can be readily integrated (ref. 3). The equations obtained treat the wave phenomena in approximate form. For a realistic treatment of ramjet transients, however, such a solution would be very complex and, hence, inconvenient to use in control-system design. In addition, interpretation of the solution in terms of changes in initial and boundary conditions may again be very difficult. For controls synthesis, therefore, it appears that further simplification of the problem merits consideration.

The simplification used in the present report is to preclude a wave solution altogether. This is done by separating the problem into two parts. In the first part, an approximation is sought for the pressure response that occurs once the initial disturbance wave has reached the sensing station. The approximation is obtained by analyzing the dynamics of the ramjet in lumped-parameter fashion, a lumped-parameter analysis being one in which the flow variables are treated as functions of time alone, rather than as functions of time and position. Such a procedure, of course, permits integration of the continuity equation (with respect to position) without recourse to Euler's equation of momentum. In the second part, the delay (dead time) between the fuel-flow disturbance and the arrival of the initial pressure wave at the sensing station is analyzed. This analysis of dead time is made on the basis of sonic propagation of the wave through steady flow.

Previous workers in the field of ramjet dynamics, of course, have used the lumped-parameter method of analysis (ref. 4, for instance). In these works, however, no consideration is given to the movement of the diffuser terminal shock that necessarily accompanies any transient condition of the gas flow. For the present analysis, it was presumed that the shock transient makes an important contribution to the dynamics of the ramjet. Accordingly, the position of the shock is treated herein as a time-dependent variable. The required definition of the shock transient is obtained from reference 5.

In the formulation of the simplified theory for response after dead time, only critical or supercritical operation of the ramjet during transients is treated. The treatment is also restricted to small transients in order to permit linearization of the equations involved. In addition to the presentation of the analysis, this report includes comparisons of the theory with experimental data obtained at the NACA Lewis laboratory. Evaluations are made for both parts of the analysis, response after dead time and dead time.

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SYMBOLS

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A	cross-sectional area of ramjet
a	speed of sound
B,C,D	steady-state quantities used in eq. (8)
K	gain of pressure in response to fuel flow
k_{sp}	gain relating change in shock position to downstream pressure disturbance
k_v	gain relating change in cold volume to change in lumped pressure
M	Mach number
p	static pressure
R	gas constant
s	Laplacian operator
T	static temperature
t	time
u	gas velocity
V	volume
w	gas flow
w_f	fuel flow

x	position coordinate in axial direction (+ downstream)
α	dead time
γ	ratio of specific heats
ρ	static density
σ	shock time constant
σ'	dimensionless shock time constant
τ_1,τ_2 τ_3,τ_4	time constants

Subscripts:

a	fuel-injection station
b	location of combustion disturbance
c	cold zone
e	exhaust-nozzle throat
h	hot zone
ss	steady state
t	total conditions
0	entering shock
1	leaving shock

RESPONSE AFTER DEAD TIME

Lumped-Parameter Analysis

The lumped-parameter concept of the ramjet can be explained by reference to figure 1. As indicated by this sketch, a "hot zone" and a "cold zone" are considered, the division being the main flameholding element of the combustor. The heat input to the engine is assumed to occur only in the hot zone. This zone, which extends downstream to the throat of the exhaust nozzle, contains a fixed volume of gas. In the cold zone, however, the volume of gas is time-dependent, since the

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terminal shock of the diffuser forms the upstream boundary. Within each zone the variation of gas temperature with position is neglected, a mean value of temperature being considered in each zone. Pressure is lumped into a mean value for the combined hot and cold zones.

In accordance with the lumped treatment of the flow variables, the principle of mass conservation states

$$w_1 - w_e = \frac{d(\rho_c V_c)}{dt} + V_h \frac{d\rho_h}{dt} \tag{1}$$

and if consideration is limited to perturbations from the steady-state condition that are small enough to neglect products of perturbation terms, the statement reduces to the linear equation

$$-\Delta w_e = \rho_c \frac{d\Delta V_c}{dt} + V_c \frac{d\Delta\rho_c}{dt} + V_h \frac{d\Delta\rho_h}{dt} \tag{2}$$

where the Δ quantities are the small perturbations, and the coefficients of the derivatives are steady-state values. (Hereafter, only Δ quantities are time-dependent.) The omission of the term Δw_1 from the equation results from restricting the analysis to ramjet operation that is critical or supercritical during transients. By use of the Laplace transformation, this linearized continuity statement takes the form

$$-\Delta w_e(s) = \rho_c s\Delta V_c(s) + V_c s\Delta\rho_c(s) + V_h s\Delta\rho_h(s) \tag{3}$$

From the equation of state $p = \rho RT$ and the assumption that in the cold zone the lumped variables of state are related isentropically, the changes in density can be related to pressure and temperature changes as follows:

$$\Delta\rho_h(s) = \frac{1}{RT_h} \Delta p(s) - \frac{p}{RT_h^2} \Delta T_h(s) \tag{4}$$

$$\Delta\rho_c(s) = \frac{1}{\gamma RT_c} \Delta p(s) \tag{5}$$

The condition of a choked exhaust nozzle permits the change in gas efflux to be related to the pressure and temperature changes in the following manner:

$$\Delta w_e(s) = \frac{w}{p} \Delta p(s) - \frac{w}{2T_h} \Delta T_h(s) \tag{6}$$

In order to define the change in the cold volume, an analysis of the shock motion is necessary. Such an analysis is presented in reference 5, where small perturbations of a normal shock in quasi-one-dimensional flow are considered. It is shown in this reference that when a disturbance in downstream pressure occurs the resultant change in shock position is related to the disturbance by a first-order lag. The time constant σ and the gain k_{sp} of this lag are expressed in terms of a dimensionless time constant σ' as follows:

$$\sigma = \frac{1}{a_{t_0}} \frac{A_0}{\left(\frac{\Delta A_0}{\Delta x_0}\right)_{ss}} \sigma'$$

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$$k_{sp} = -\frac{\gamma + 1}{4\gamma} \frac{a_0}{a_{t_0}} \frac{1}{p_0 M_0} \frac{A_0}{\left(\frac{\Delta A_0}{\Delta x_0}\right)_{ss}} \sigma'$$

(Negative values of k_{sp} denote upstream movement of the shock.) The dimensionless time constant σ' depends only on the steady-state value of the shock Mach number M_0, being defined in the following manner:

$$\sigma' = \frac{2\left(\frac{\gamma + 1}{\gamma - 1}\right) M_0 \left(1 + \frac{\gamma - 1}{2} M_0^2\right)^{1/2}}{1 + \frac{\gamma^2 + 1}{\gamma - 1} M_0^2}$$

With this information from reference 5, the change in the cold volume can be related to the lumped pressure by the equation

$$\Delta V_c(s) = k_v \frac{1}{1 + \sigma s} \Delta p(s) \tag{7}$$

where

$$k_v = -A_0 k_{sp}$$

Equations (4) to (7) can now be combined with the continuity expression of equation (3) to obtain a single equation containing only the changes in pressure and hot-zone temperature as variables. This equation, together with the assumption that hot-zone temperature and fuel flow are related quasi-statically, will yield the following transfer function for pressure to fuel flow:

$$\frac{\Delta p(s)}{\Delta w_f(s)} = K \frac{1 + (2B+\sigma)s + 2B\sigma s^2}{1 + (B+C+D+\sigma)s + (B+C)\sigma s^2} \tag{8}$$

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where

$$B = \frac{pV_h}{wRT_h} \qquad\qquad D = \frac{k_v p^2}{wRT_c}$$

$$C = \frac{pV_c}{\gamma wRT_c} \qquad\qquad K = \frac{p}{2T_h}\left(\frac{\Delta T_h}{\Delta w_f}\right)_{ss}$$

4442 A more convenient form of the transfer function is obtained when equation (8) is factored as follows:

$$\frac{\Delta p(s)}{\Delta w_f(s)} = K\,\frac{(1 + \tau_1 s)(1 + \tau_2 s)}{(1 + \tau_3 s)(1 + \tau_4 s)} \tag{9}$$

where

$$\tau_1 = \sigma$$

$$\tau_2 = 2B$$

$$\tau_3 = \frac{2\sigma(B+C)}{(B+C+D+\sigma) - \sqrt{(B+C+D+\sigma)^2 - 4\sigma(B+C)}}$$

$$\tau_4 = \frac{2\sigma(B+C)}{(B+C+D+\sigma) + \sqrt{(B+C+D+\sigma)^2 - 4\sigma(B+C)}}$$

According to lumped-parameter considerations, therefore, the response of pressure to fuel flow in the ramjet is a second-order lead-lag characterized by the time constants τ_1, τ_2, τ_3, and τ_4. These time constants, in turn, are functions of the quantities B, C, and D and the shock time constant σ. The quantities B and C can be thought of as the "filling times" of the hot and cold volumes, respectively, as they define the times required to fill these volumes with gas at the steady-state volume flow rate. The quantity D and the time constant σ represent the contribution of the shock to the dynamics of the engine.

These filling times and shock-associated terms can be calculated for a given ramjet configuration once the steady-state flight conditions and engine operating point are specified. The simplified description of the steady flow necessary for these calculations is readily obtainable; either the well established theory for steady flow or experimental data can be used for this purpose. For computing the shock time constant σ (as well as the gain k_v involved in the quantity D) the curve shown

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in figure 2 is useful. This curve, which is reproduced from reference 5, gives the value of the dimensionless shock time constant σ' for shock Mach numbers ranging in value from 1.0 to 3.0 ($\gamma = 7/5$).

It should be emphasized that the transfer function derived in this section must be combined with a term for dead time to obtain the complete transfer function for the ramjet. The complete transfer function is

$$\frac{\Delta p(s)}{\Delta w_f(s)} = Ke^{-\alpha s} \frac{(1 + \tau_1 s)(1 + \tau_2 s)}{(1 + \tau_3 s)(1 + \tau_4 s)}$$ 4442

where α is the dead time at the station of interest. The discussion of dead time follows the experimental corroboration of the foregoing analysis.

Comparison of Theoretical and Experimental Responses

The lumped-parameter theory for response after dead time has been compared with the experimentally determined responses of two ramjets. These engines were both full-scale and representative of current ramjet design. They were considerably different, though, in size, configuration, and flight specifications and thus afforded a rather extensive check on the theory. The dynamic behavior of these engines had been well defined experimentally. Test programs had been conducted in which steps or sinusoidal variations in fuel flow were imposed by means of specially designed electrohydraulic servo systems. The resultant response of pressure was measured at several engine locations with instrumentation suitable for recording pressure fluctuations up to 100 cycles per second in frequency. During the tests, the ramjets were operated in supersonic airstreams generated by free-jet facilities.

In the calculations of the theoretical responses, the steady-state descriptions required were obtained by means of one-dimensional theory in conjunction with the available experimental data. For simplicity, the assumption was made that the entire heat input to an engine occurred at its main flameholding element. In the computations of the lumped values of flow variables, the variables were first determined as functions of position in the region being considered, and then the means were taken by a volume-weighting process. In order to determine the values of the shock time constant σ and the gain k_v for one engine, designated ramjet I, the parameters involved in these quantities were averaged over the region traversed by the shock. This was necessary because rather large movements of the shock were being treated. For the other engine, designated ramjet II, the shock terms could be computed from the initial condition, since the shock travel was small.

The responses calculated for the two engines are compared with experimental data in figure 3. The comparison, which is presented in figures 3(a) and (b) for ramjet I and in figure 3(c) for ramjet II, is made on the basis of frequency response; amplitude ratio and phase shift are shown as functions of frequency. The amplitude ratios are given in normalized form, that is, they have been adjusted to a steady-state gain of unity. The responses from the lumped-parameter theory are shown in the figure by the solid curves, while the experimental data are represented by symbols. The straight-line approximations of the amplitude curves are given as dashed lines, and the break frequencies $(1/2\pi\tau)$ associated with the calculated time constants are indicated.

From these break frequencies it is evident that the dynamics of the shock play an important role in the theory formulated. For instance, the time constant τ_1, which is equivalent to the shock time constant σ, contributes strongly to the shape of the theoretical amplitude curve for each ramjet.

The experimentally determined responses include the effects of dead time, of course. Since the amplitude component of dead time is always unity, the amplitude data afford a direct means of evaluating the theory for response after dead time. In plotting the phase-shift curves, however, it was necessary to combine the phase characteristics of the lumped-parameter theory with those of the appropriate dead times. The values of dead time used for this purpose were determined in the step-response tests. The phase curves without this adjustment are also included in the figures.

The data for ramjet I in figure 3(a) are presented for one location of the pressure tap but for fuel-flow sinusoids of two different amplitudes. The data shown by the circular symbols were obtained with the fuel-flow amplitude as small as practical; the square symbols represent data from a sinusoid of large amplitude. The figure shows the experimental results for both sinusoids are described reasonably well by the theoretical curves. In fact, the change in fuel-flow amplitude produced no sizable effect on either the amplitude or phase characteristics of the pressure response. On the basis of this figure, therefore, the linearization performed in the analysis by the assumption of small transients does not impair the usefulness of the theory. The theory may be used to predict the approximate response for large as well as small transients.

Any difference in response from one station to another is not considered, of course, in the lumped-parameter analysis. The extent of error in such a treatment can be seen in figure 3(b). The shaded band in the figure indicates the effect of pressure-sensor location on the experimental results for ramjet I. This effect is shown only for the amplitude characteristics because the phase effect was predominantly due to dead-time changes. The width of the shaded band at a given frequency denotes the maximum variation of the amplitude-ratio data for stations throughout the ramjet. (The variation appeared to be completely random.) The figure indicates that the theory provides a reasonable approximation to the response at any station in the ramjet. Further evidence of this is shown in figure 3(c), where the amplitude data for ramjet II are presented for two stations in the engine. In this figure the curve for the theoretical response gives a good approximation of the experimental data for both stations over most of the frequency range.

In summary of the experimental corroboration, figure 3 indicates that the lumped-parameter theory reasonably approximates the response of ramjet pressures to fuel flow, exclusive of dead time, of course. Evidence is given that the theory may be used for pressures throughout the engine and for large as well as small disturbances in fuel flow. It is believed that the theory is sufficiently accurate to be useful in the design of ramjet controls.

DEAD TIME

The dead time in the response of ramjet pressures to fuel flow consists of two delays: first, the time required for the fuel (injected at a new rate) to travel from the injection station to its place of burning in the combustion chamber, and second, the time spent by the resultant pressure wave in propagating from the combustion disturbance to the pressure-sensing station. There appears to be no way to locate, by analytical means, the place in the combustion chamber where combustion is disturbed and the pressure wave originates. Experimental data indicate the location to be some distance downstream of the main flameholder; however, the distance depends on the initial state of the gas flow, the size of the fuel-flow change, and whether the fuel flow was increased or decreased. Fortunately, the error caused by this uncertainty is usually small in comparison to the dead times being calculated. Therefore, an approximate location may be used. For ramjets I and II the distance was assumed to be one-third of the way from the main flameholder to the nozzle throat.

Once an approximate location has been selected for the combustion disturbance, the calculation of dead time becomes essentially a problem of determining the gas velocities and the sonic velocities in the engine for steady-state operation. The fuel is considered to travel at the velocities of the gas, and the pressure wave is assumed to propagate at sonic velocities relative to the gas. The assumption that these velocities are those of the steady state is, in general, a small-perturbation treatment; for the general problem relating to control response, of course, the fuel-flow change may be preceeded by transient flow conditions.

Dead time may be defined in terms of the steady-state values of the gas velocity u and sonic velocity a as follows:

$$\alpha_x = \int_{x_a}^{x_b} \frac{1}{u}\, dx \pm \int_{x_b}^{x} \frac{1}{a \pm u}\, dx \tag{10}$$

where α_x is the dead time at station x, x_a is the fuel injection station, and x_b is the station of combustion disturbance. The velocity u is the component in the x-direction, and the value of x is considered to increase in the downstream direction. The plus signs apply when x is greater than x_b, and the minus signs are to be used when x is less than x_b.

Methods for calculating the velocities u and a for steady flow are well known and need not be discussed herein. The tables of reference 6 are very convenient for this purpose. Often the theory for these calculations can be supplemented by experimental data. This was done in the velocity calculations for ramjets I and II; some experimental pressure and temperature data were used in conjunction with one-dimensional theory. In addition, as in the calculations for the lumped-parameter response, it was assumed for simplicity that steady-state combustion occurred only at the main flameholders.

The dead times calculated for ramjets I and II are compared in figure 4 with those determined experimentally during the step-response tests. The dead times are given for the region of each engine downstream of the normal shock. The calculated variation of dead time in this region is shown by the curve, and the experimental data are represented by the symbols. For ramjet II the data are given for both a step increase and a step decrease in fuel flow. For ramjet I there was no separation of the data with the direction of the step.

As can be seen in figure 4, the calculated dead times agree reasonably well, in general, with the experimental values. Some deviation, however, can be seen for ramjet II (fig. 4(b)). For this engine, in which the data depend on the direction of the fuel-flow step, the calculated curve agrees better with the dead times measured for a step increase than with those measured for a step decrease. The deviation of the curve from the step-decrease data is most pronounced for stations downstream of the flameholder. For stations upstream of the flameholder, the curve and the step-decrease data are in fair agreement.

CONCLUDING REMARKS

For critical or supercritical operation of the ramjet, it appears that disturbances in fuel flow provoke pressure responses that, exclusive of dead time, can be described approximately by a second-order lead-lag function obtained from lumped-parameter considerations. The approximation is considered sufficiently accurate to be useful to the controls designer. The time constants of the theoretical lead-lag are completely determined by the geometry of the ramjet and a simplified description of the steady-flow conditions preceeding the transient. These time constants, therefore, can be readily calculated with the use of steady-state data or well established flow theory.

The time constants are composed of terms that describe the filling times of the ramjet and the dynamic effects of the diffuser terminal shock. Filling time refers to the time required to fill a significant volume in the engine at the volume flow rate that exists in the steady state. The dynamic effects of the shock result from the transient in shock position that accompanies the pressure response, the shock position being dynamically related to the pressure by a first-order lag.

The complete transfer function relating a ramjet pressure p to fuel flow w_f is obtained by multiplying the second-order lead-lag function by the term for the dead time at the station involved. The complete transfer function is

$$\frac{\Delta p(s)}{\Delta w_f(s)} = Ke^{-\alpha s}\frac{(1 + \tau_1 s)(1 + \tau_2 s)}{(1 + \tau_3 s)(1 + \tau_4 s)}$$

where α is the dead time, τ_1, τ_2, τ_3, and τ_4 are time constants, K is the gain, and s is the Laplacian operator. The dead time can be calculated once the place of the combustion disturbance has been located with reasonable accuracy. The fuel can be considered to travel from the injector to this place of disturbance at the steady-state gas velocities; the resultant pressure wave can be assumed to propagate relative to the gas at the steady-state sonic velocities.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, August 16, 1957

REFERENCES

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1. Rudinger, George: Wave Diagrams for Nonsteady Flow in Ducts. D. Van Nostrand Co., Inc., 1955.

2. Shapiro, Ascher H.: The Dynamics and Thermodynamics of Compressible Fluid Flow. Vol. II. The Ronald Press Co., 1954.

3. Aschenbrenner, Josef (J. Lotsof, trans.): Non-Stationary Gas Flow in Ducts of Variable Cross Section. Cornell Aero. Lab. Trans., Oct. 1946.

4. Bodner, V. A. (A. Petren, trans.): Ramjet Automation and Regulation. Automatica Aviatsionna Dvigateli, Moscow, 1956. (Marquardt Aircraft Co. trans.)

5. Hurrell, Herbert G.: Analysis of Shock Motion in Ducts During Disturbances in Downstream Pressure. NACA TN 4090, 1957.

6. Ames Research Staff: Equations, Tables, and Charts for Compressible Flow. NACA Rep. 1135, 1953. (Supersedes NACA TN 1428.)

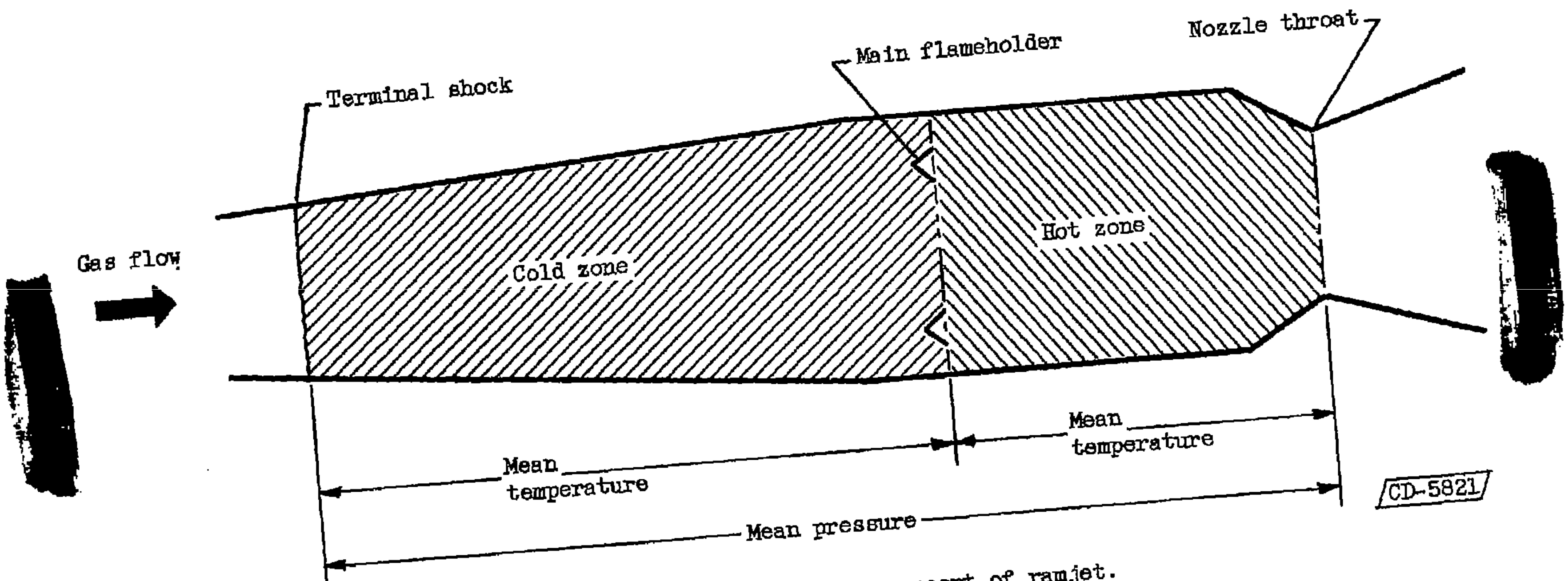

Figure 1. - Lumped-parameter concept of ramjet.

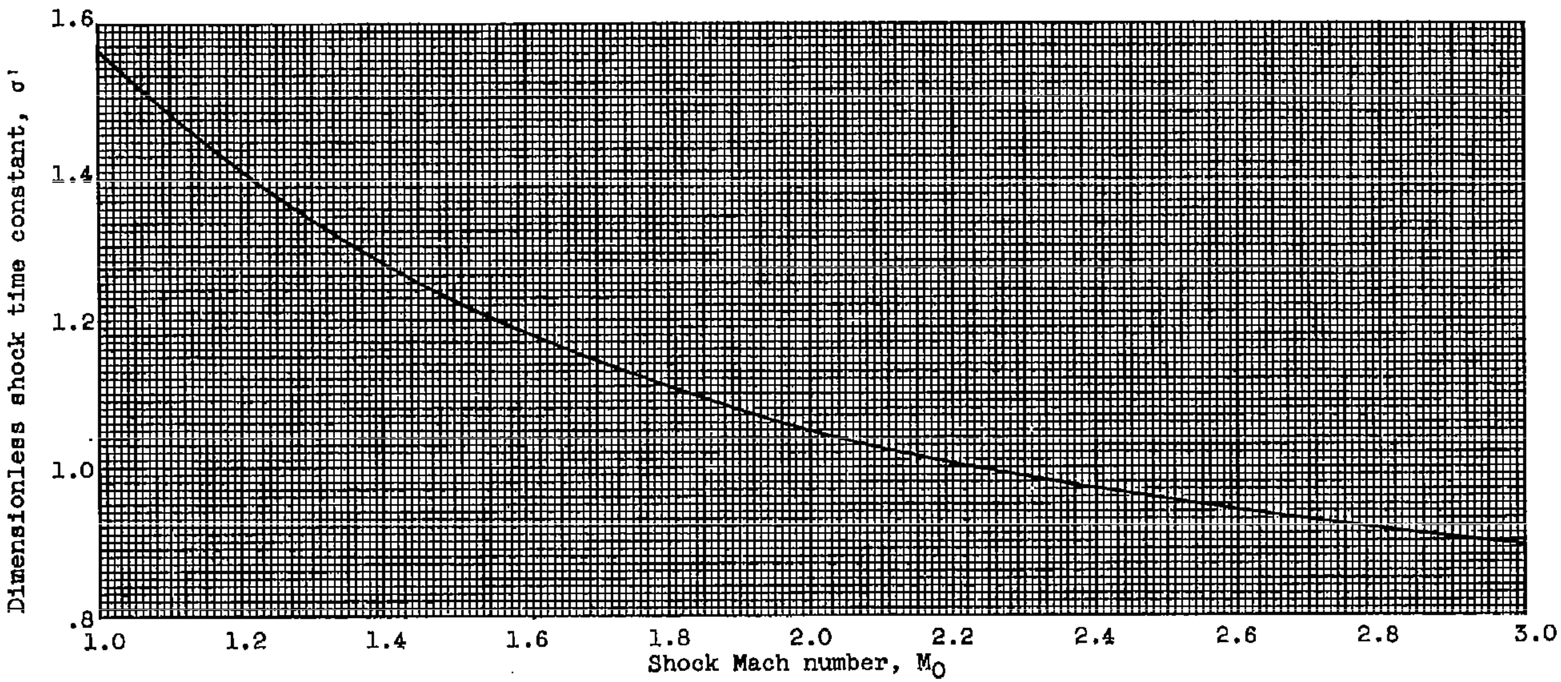

Figure 2. - Variation of dimensionless shock time constant with shock Mach number. Ratio of specific heats, 1.4.

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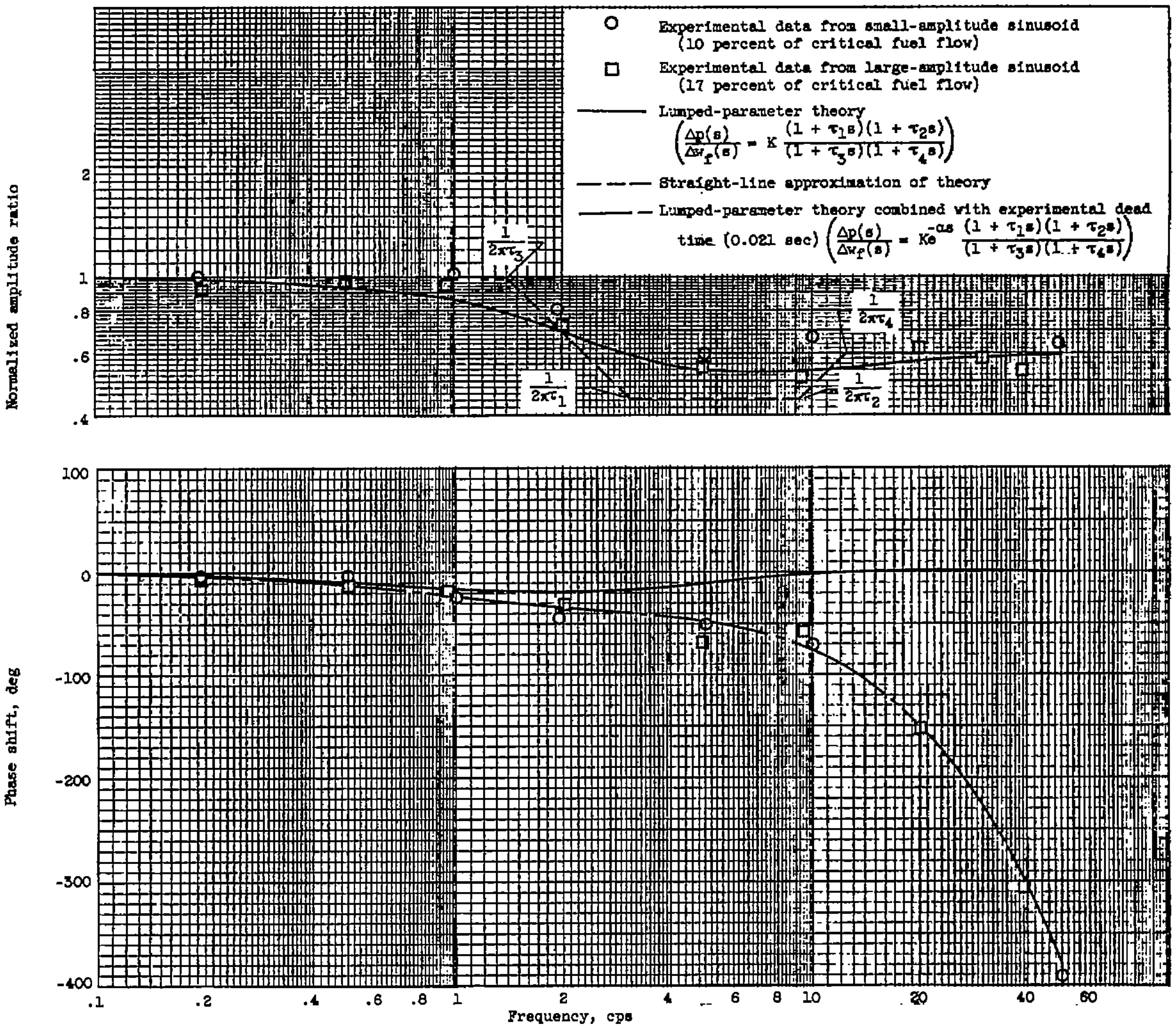
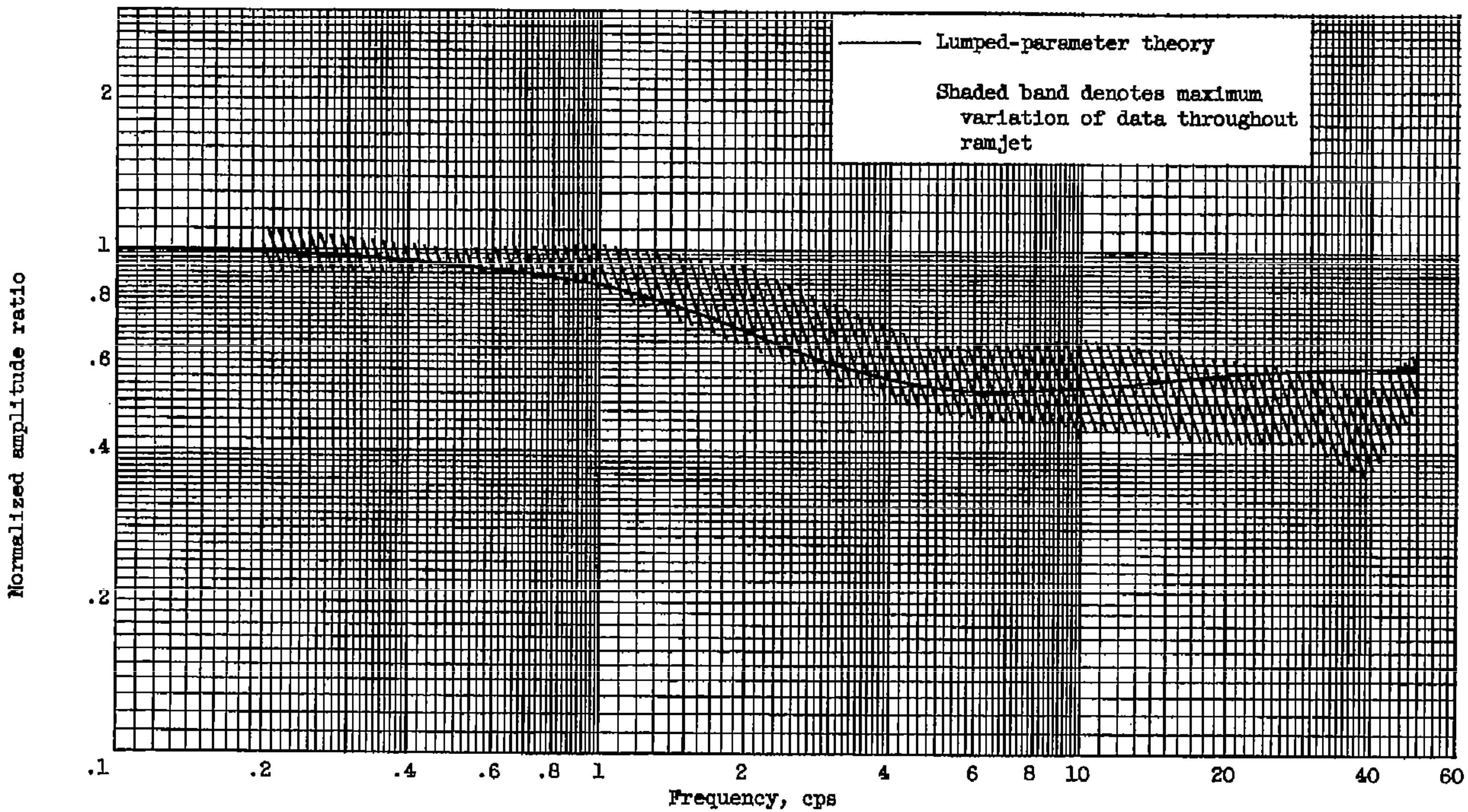


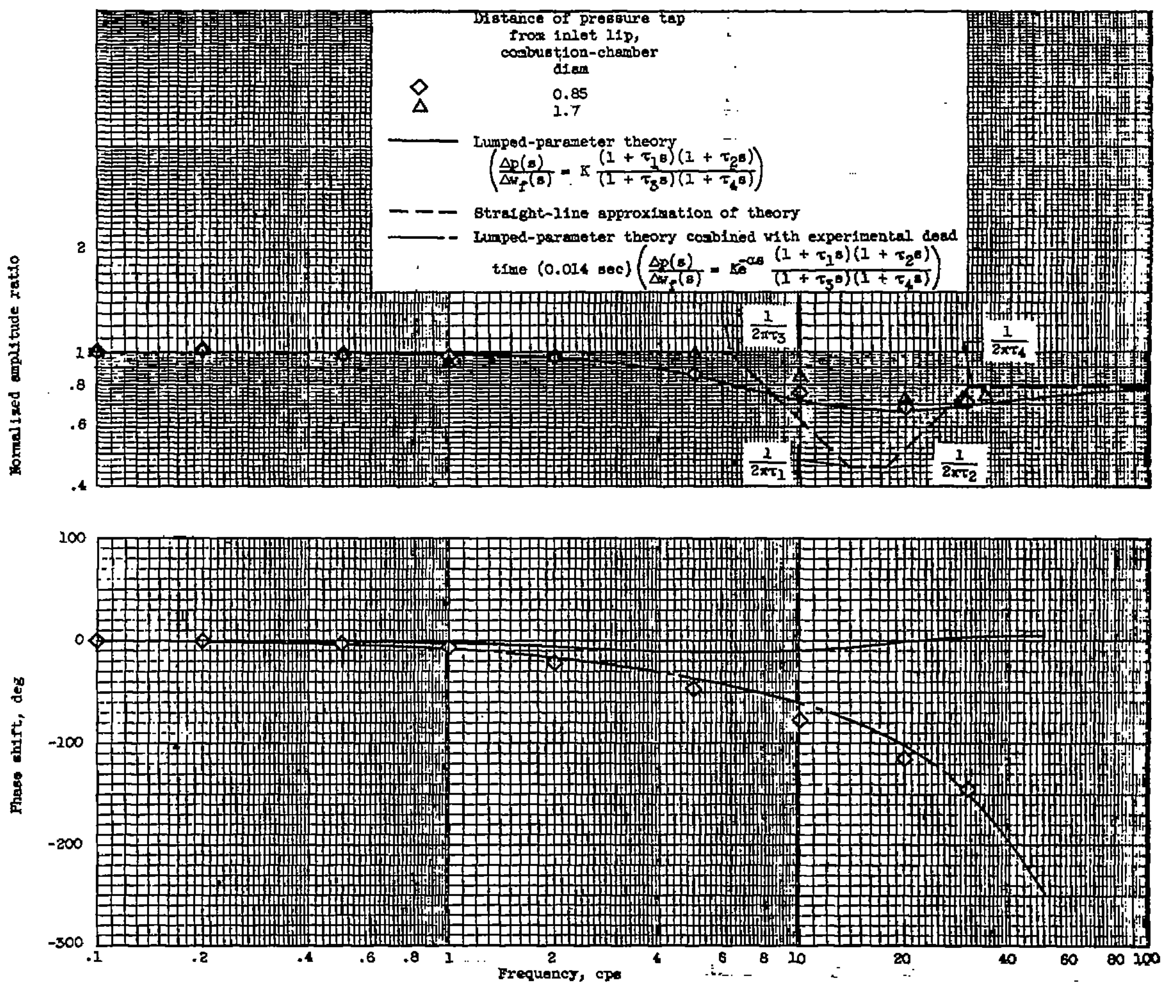
(a) Ramjet I; pressure tap near diffuser exit.

Figure 3. - Comparison of theoretical and experimental responses of pressure to fuel flow after dead time. Diffuser total-pressure recovery, 95 percent of critical.



(b) Ramjet I; range of pressure tap locations.

Figure 3. - Continued. Comparison of theoretical and experimental responses of pressure to fuel flow after dead time. Diffuser total-pressure recovery, 95 percent of critical.



(c) Ramjet II. Amplitude of fuel-flow sinusoid, 12.5 percent of critical fuel flow.

Figure 3. - Concluded. Comparison of theoretical and experimental responses of pressure to fuel flow after dead time. Diffuser total-pressure recovery, 95 percent of critical.

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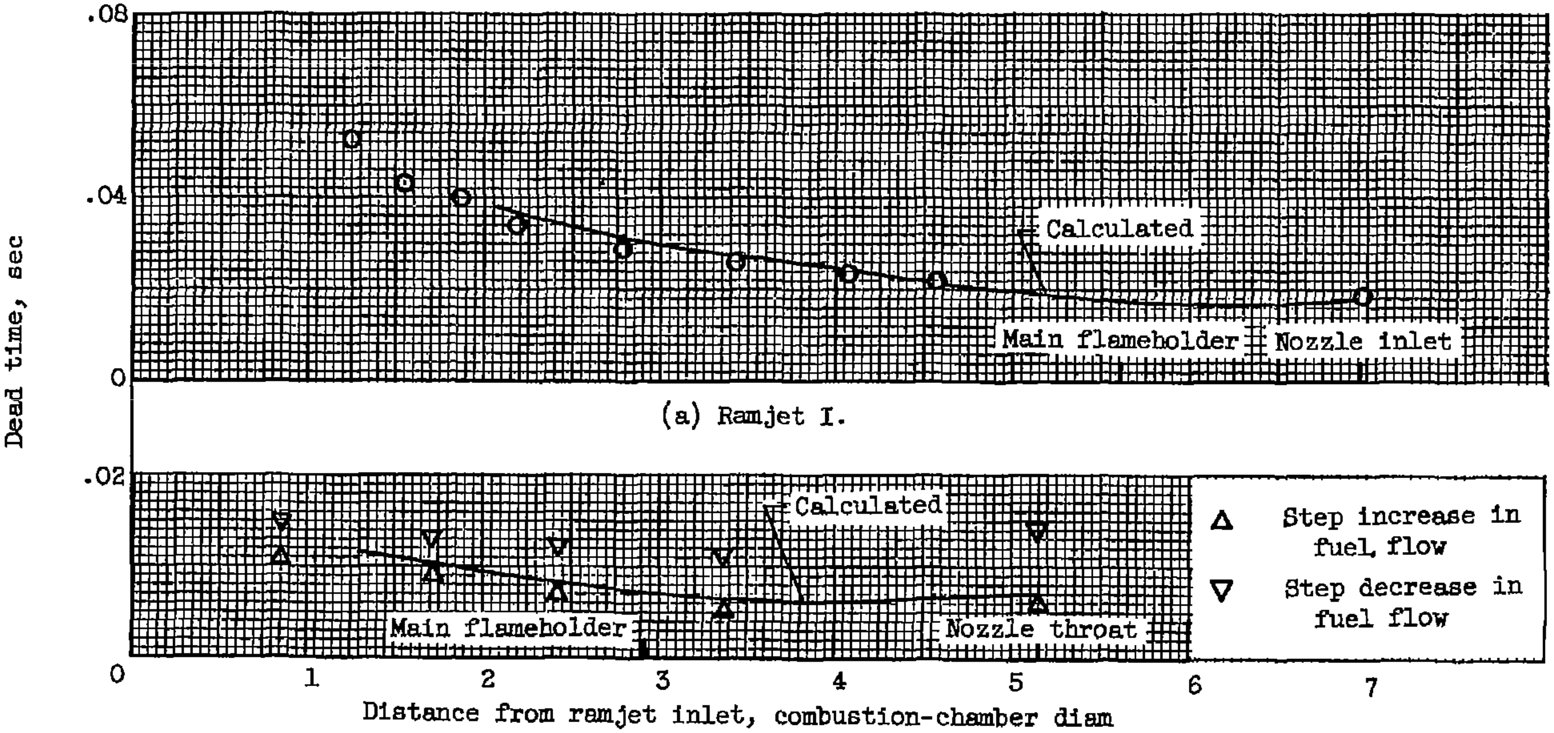

(b) Ramjet II.

Figure 4. - Comparison of calculated and experimental dead times of pressure to fuel flow. Diffuser total-pressure recovery, 98 percent of critical; step change in fuel flow, 10 percent of critical fuel flow.